Design of Stirling Engine

The compression ratio, or volume ratio *VR*, is defined as the gas volume with the piston at top dead centre (TDC) divided by the gas volume with the piston at bottom dead centre (BDC).It is typically very low in LTD engines due to the large overall gas volume that is generally required for these engines to operate. Kolin [1] describes a 'rule of the thumb' for approximating the ideal volume ratio which depends on the temperature difference of the engine in question. The formula states:

$$V_R = \left(1 + \frac{\Delta T}{1100}\right) \tag{1}$$

The phase angle is almost always fixed at 90° however this is not necessarily the optimum value, as indicated by the graph in Figure 1 which describes a numerical analysis of a certain Philips Stirling engine.

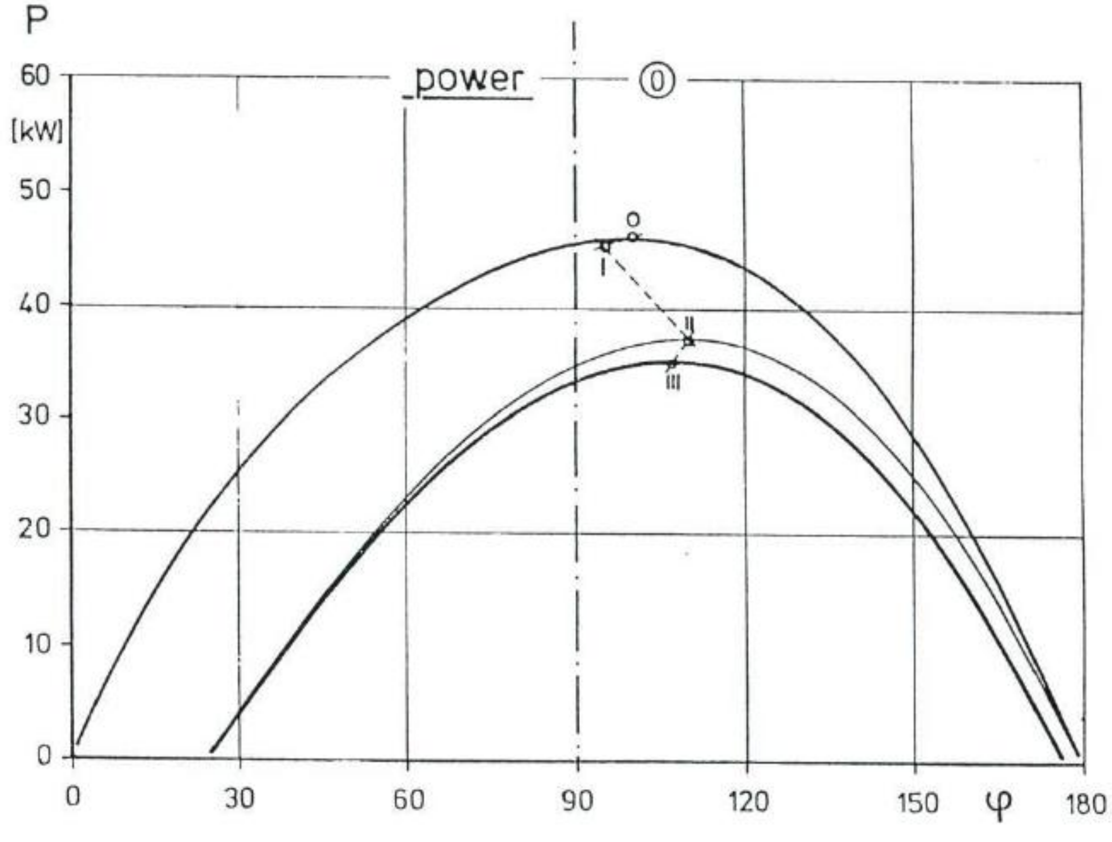

Figure 1: Graph of power output vs. phase angle for 3 different dead volumes [2]

One such estimate is made using the Beale number, an empirically derived number named after William Beale, inventor of the free-piston Stirling engine. He noted that the performance of many Stirling Engines tended to conform to the following simple equation relating indicated power, *Po* (W), to pressure *p* (bar), operating frequency *f* (Hz) and expansion space volume *Ve* (cm3) with the Beale number *Bn*.

$$P_o = B_n pfV_e \qquad (2)$$

Figure 2 shows a graph plotted by measuring data from many Stirling Engines. The solid line in the middle is typical of most Stirling Engines while the upper and lower lines denote unusually high or low performing engines. Based on this graph, with a heater temperature of 90°C (363K), it would be reasonable to expect a Beale number in the range of 0.002 to 0.008. For further calculations a Beale number of 0.005 is used.

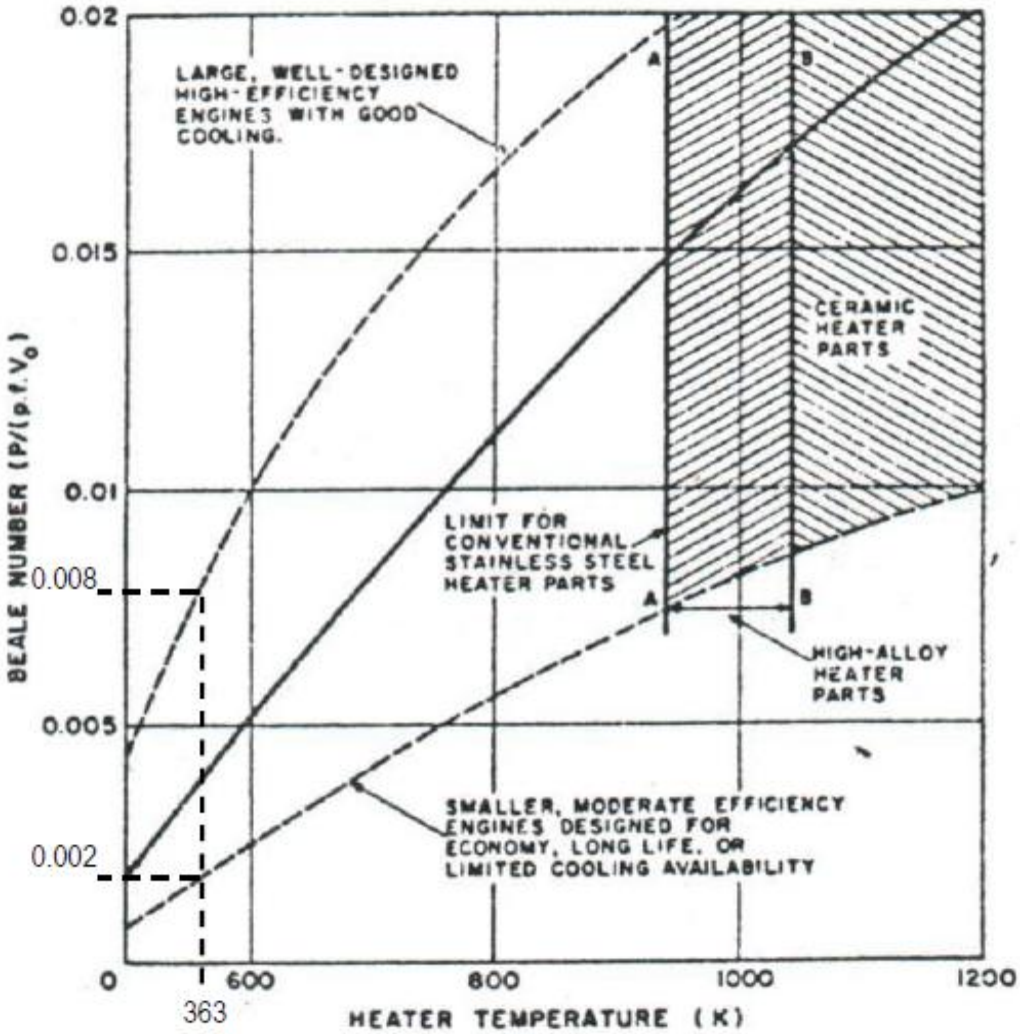

Figure 2: Graph of Beale number vs. heater temperature for a range of engines [2]

At this point, Equation 2 can be rearranged and solved to find an estimate of the expansion space volume needed to reach the target power output based on the Beale number. Choosing an average Beale number of 0.005 and solving for a nominal output of 500 W:

$$V_e = \frac{P_o}{B_n\, pf} = \frac{500}{0.005 \times 10 \times 2} = 5000\ cm^3 = 5l$$

Which means the piston must sweep about 5 litres of volume. From Equation 1 the volume ratio *VR* can be calculated and this will then give an indication of the size of displacer chamber needed.

$$V_R = \left(1 + \frac{\Delta T}{1100}\right) = \left(1 + \frac{70}{1100}\right) = 1.06$$

And

$$V_R = \frac{V_{\max}}{V_{\min}} = \frac{V_c + V_e}{V_c}$$

Therefore

$$V_c = \frac{V_e}{V_R - 1} = \frac{5000}{0.06} = 83{,}000 = 83l$$

So for a maximum expected temperature difference of 70°C the volume of the displacer chamber (the compression space) should be about 83 litres. The volume of the compression space is fixed once chosen, and since the engine also should run on temperatures less than 70°C, a value of around 130 litres is more appropriate. This allows the engine to run ideally on temperature differences of around 40-50°C, and if the stroke is reduced (thus reducing both *VR* and *Ve*) then the temperature difference should be able to drop to around 20°C while still maintaining a close to optimal volume ratio. This does leave the engine somewhat short on compression towards the higher end of its operating temperature range, however in the interests of researching a low temperature differential engine it was considered more beneficial to skew the design in this direction and investigate lower temperature differential operating limits.

After examining available sizes in steam pipes and other available pipes in large diameters it was found the biggest readily available diameter size was a nominal bore of 800 mm (813 od x 10.0 wt). This spiral pipe was chosen for use as displacer chamber, giving an internal diameter *dc* of 793 mm. To calculate the required length *Lc* of the displacer chamber for a swept volume *Vc* of 130 litres:

$$V_c = \frac{1}{3}\pi\left(\frac{d_c}{2}\right)^2 L_c \qquad (3)$$

Therefore

$$L_c = \frac{3V_c}{\pi\left(\frac{d_c}{2}\right)^2} = \frac{3 \times 0.13}{\pi\left(\frac{0.793}{2}\right)^2} = 0.79\, m \approx 800\ mm$$

Hence a value of 800 mm was used for the length of the displacer chamber. Note the factor of three used in the equation is because the displacer and heat exchangers take up a third of the volume each, leaving one third of the remaining volume as swept volume.

The cylinder in which the power piston resides needs to be a honed tube, i.e. perfectly round and smooth inside. Available sizes of honed tube are somewhat limited, with 228 mm being the largest readily available size. A cylinder of this diameter has a cross-sectional area of 0.0408 m2, meaning a stroke of only 122 mm is needed to obtain the desired expansion space volume of 5 litres. It was decided to use a maximum possible stroke length of 150 mm to allow some room for adjustment on either side. This would allow the use of higher temperatures through a higher volume ratio.

A stroke length of 150 mm dictates the length of the cylinder (stroke length + height of piston) and the size of the chamber in which the crankshaft resides. This chamber is also made of steam pipe similar to the displacer chamber. A pipe with a nominal bore of 450 mm was chosen for this as it provides adequate clearance for the crank.

Reference:

1. **Kolin, Ivo.** *Stirling Motor - History, Theory, Practice.* Dubrovnik : Zagreb University Publications, Ltd., 1991.

2. **Martini, W. R.** *Stirling Engine Design Manual.* Richland : Martini Engineering, 1983.